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PROPAGATION OF ACOUSTIC SIGNALS IN A TWO-PHASE MEDIUM OF SLUG STRUCTURE

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The theoretical description of the characteristics of propagation of acoustic signals in a gas–liquid mixture of plug-train structure has been so far based on relatively simple models that assume either a periodic slug arrangement or the existence of a weak irregularity in the sizes of liquid plugs and gas slugs.

In the present paper, we propose using a random telegraphic process to define the acoustic characteristics of a two-phase medium, which change successively from slugs to plugs. The essentially one-dimensional nature of plug-train flow allows us to apply the developed methods of the theory of one-dimensional unordered media and obtain a closed system of equations for description of the statistical characteristics of an acoustic field. The solution of these equations permits one to analyze a number of effects (passage through a layer and statistical parametric resonance) and demonstrate the distinctive features of wave propagation in a mixture of slug structure.

Gas- and steam–liquid mixtures can move at various regimes (bubble, slug, core ones, etc.) [1, 2]. At present, acoustic-wave propagation in a liquid with bubbles has been studied in sufficient detail [2, 3]. Less attention has been given to the description of acoustic perturbations in a mixture of slug structure. At the same time, the essentially one-dimensional character of this flow allows one to use the developed procedures of the theory of one-dimensional unordered systems and to obtain, in a number of cases, an exact and, therefore, far more detailed description compared with analysis of wave propagation in bubble media.

We shall use the slug-flow model by Lezhnin [4], which also finds application in analysis of one-dimensional unordered media [5]. Figure 1 shows a diagram of the slug-flow structure. The sizes of alternating gas (steam) l_g and liquid l_f layers are variable. The gas is considered ideal with specific heat ratio γ. Fluid friction against the channel walls and interphase friction are ignored. A plane wave $P_{in}\exp[-ik_1(z-L)-i\omega t]$ ($k_1=\omega/c_1$), where ω is the wave frequency, falls from the right of a homogeneous medium with density ρ_1 and sound velocity c_1 on a two-phase mixture. The scattered wave $R\exp[ik_1(z-L)-i\omega t]$ is in the same area. The transmitted wave $P_{tr}\exp[-ik_3(z-L_0)-i\omega t]$ ($k_3=\omega/c_3$) is in the homogeneous medium with parameters ρ_3 and c_3 to the left of the two-phase flow area.

The theory of propagation of acoustic pulses in a periodical slug structure with allowance for nonlinear distortions has been developed in [6]. Taking into account a weak irregularity in slug and plug sizes allowed Lezhnin et al. [4] to obtain small corrections to the law of wave dispersion. Analysis of an essentially heterogeneous situation [5] is an interesting but special problem: the search for acoustic analogues of the Anderson localization in one-dimensional systems.

Acoustic Model of Slug Flow. The easy-to-grasp pattern of acoustic perturbations, which have the shape of plane waves propagating in opposite directions within each layer and scattering at interfaces (with the conditions of pressure P and velocity v continuity satisfied), can also be described by the continuity equations:

$$\rho_0(z)\frac{\partial v}{\partial t}=-\frac{\partial P}{\partial z},\quad \frac{\partial \rho}{\partial t}+\rho_0(z)\frac{\partial v}{\partial z}=0,\quad P=c_0^2(z)\rho. \tag{1}$$

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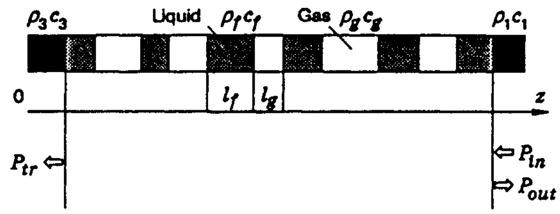

Fig. 1

Here the density $\rho_0(z)$ and the sound velocity $c_0(z)$, which vary from slug to plug, can be defined by introducing a random telegraphic process $s(z)$:

$$\rho_0(z) = [(\rho_f + \rho_g) + (\rho_f - \rho_g)s(z)]/2, \quad c_0(z) = [(c_f + c_g) + (c_f - c_g)s(z)]/2,$$

where ρ_f and ρ_g are the densities and c_f and c_g are the sound velocities in a liquid plug and in a gas slug, respectively; $s(z)$ is a random telegraphic process which takes on the values $+1$ and -1. Let us give the statistical characteristics of the telegraphic process [7] as applied to the problem considered.

The probability density that the liquid plug is at the point z' has the form

$$P_+(z', z) = \frac{l_f}{l_f + l_g} + \exp\Big[-\Big(\frac{1}{l_f} + \frac{1}{l_g}\Big)(z' + z)\Big]\Big[\Big(\frac{l_g}{l_f + l_g}\Big)\delta_{+z} - \Big(\frac{l_f}{l_f + l_g}\Big)\delta_{-z}\Big],$$

and the probability density that the gas slug is at the same point is of the form

$$P_-(z', z) = \frac{l_g}{l_f + l_g} - \exp\Big[-\Big(\frac{1}{l_f} + \frac{1}{l_g}\Big)(z' + z)\Big]\Big[\Big(\frac{l_g}{l_f + l_g}\Big)\delta_{+z} - \Big(\frac{l_f}{l_f + l_g}\Big)\delta_{-z}\Big].$$

Here $\delta_{+z} = 1$ if the liquid plug is at the point z, and $\delta_{+z} = 0$ if the slug is there; similarly, $\delta_{-z} = 1$ for the gas slug and $\delta_{-z} = 0$ for the liquid plug (evidently, $P_+ + P_- = 1$), l_f and l_g are the average sizes of liquid plugs and gas slugs. The conditional probabilities that a continuous liquid $P_f(l) = (1/l_f)\exp(-l/l_f)$ or gas $P_g(l) = (1/l_g)\exp(-l/l_g)$ layer is in the interval l define the distribution of liquid plugs and gas slugs. We can now determine the average characteristics of the medium's parameters:

$$\langle s\rangle = \frac{l_f - l_g}{l_f + l_g}, \quad \langle \rho_0(z)\rangle = \rho_f\,\frac{l_f}{l_f + l_g} + \rho_g\,\frac{l_g}{l_f + l_g}, \quad \langle c_0(z)\rangle = c_f\,\frac{l_f}{l_f + l_g} + c_g\,\frac{l_g}{l_f + l_g}.$$

The problem of wave scattering over the layer can be reduced to the boundary-value problem for acoustic perturbations inside the medium. Indeed, according to (1), the acoustic perturbations at the frequency ω in slug flow are described by the system

$$\frac{dv}{dz} = -\frac{i\omega}{\rho_0(z)c_0^2(z)}\,P, \quad \frac{dP}{dz} = i\rho_0(z)\omega v \tag{2}$$

and satisfy boundary conditions of the kind $P(L_0) + z_3 v(L_0) = 0$ and $P(L) - z_1 v(L) = 2$, where L_0 is a coordinate of the left boundary of the slug, L is a coordinate of the right boundary, $z_1 = \rho_1 c_1$, $z_3 = \rho_3 c_3$, and the incident-wave amplitude P_{in} is chosen as a measuring unit.

Equations of Invariant Immersion. Satisfaction of the principle of dynamic causality is one of the indisputable criteria for applicability of the present-day methods to analysis of stochastic equations. Unfortunately, this criterion is not valid for the above-formulated boundary-value problem, because P and v in $(\cdot)z$ are functions of all the values of $s(z)$ in the interval $L_0 < z < L$; moreover, even the boundary conditions are a functional of the field $s(z)$. In view of this, we shall use, according to [8], the invariant-immersion procedure which permits one to go from the boundary-value problem to the problem with initial data (Cauchy problem). For the latter problem, the principle of dynamic causality is satisfied; therefore, under certain assumptions on the statistics of the process $s(z)$ it turns out to be possible to derive an equation for the probability density of solution of problem (2).

The nondependence (invariance) of the coefficients of the equation and boundary conditions (2) on the layer thickness allows us to obtain the equation for the pressure field $P(z, L)$ using the immersion parameter L [8]:

$$\frac{\partial P(z,L)}{\partial L} = -iP(z,L)\Big[-\frac{k_2 z_2}{z_1} + \frac{k_2(z_2^2 - z_1^2)}{2z_1 z_2}P(L,L)\Big],$$
$$\frac{dP(L,L)}{dL} = -2i\frac{k_2 z_2}{z_1}(1 - P(L,L)) - i\frac{k_2(z_2^2 - z_1^2)}{2z_1 z_2}P^2(L,L), \quad P(L_0, L_0) = \frac{2z_3}{z_1 + z_3}. \tag{3}$$

Here $P(L, L)$ is the pressure at the right boundary and $P(L_0, L_0)$ is the pressure at the left boundary when the layer thickness tends to zero, $k_2(z) = \omega/c_0(z)$, and $z_2(z) = \rho_0(z)c_0(z)$.

It is convenient to select explicitly the average and fluctuation components in the coefficients of this equation:

$$\frac{k_2 z_2}{z_1} = k_1\Big[\frac{\langle\rho_0(z)\rangle}{\rho_1} + \frac{(\rho_f - \rho_g)}{2\rho_1}(s - \langle s\rangle)\Big],$$
$$\frac{k_2(z_2^2 - z_1^2)}{2z_1 z_2} = k_1\Big\{\Big[\frac{\langle\rho_0(z)\rangle}{\rho_1} - \rho_1 c_1^2\Big\langle\frac{1}{\rho_0 c_0^2}\Big\rangle\Big] + \Big[\frac{(\rho_f - \rho_g)}{\rho_1} - \rho_1 c_1^2\Big(\frac{1}{\rho_f c_f^2} - \frac{1}{\rho_g c_g^2}\Big)\Big](s - \langle s\rangle)\Big\},$$
$$\langle\rho_0\rangle = \rho_f\frac{l_f}{l_f + l_g} + \rho_g\frac{l_g}{l_f + l_g}, \quad \Big\langle\frac{1}{\rho_0 c^2}\Big\rangle = \frac{1}{\rho_f c_f^2}\frac{l_f}{l_f + l_g} + \frac{1}{\rho_g c_g^2}\frac{l_g}{l_f + l_g}.$$

Weak Fluctuations. We shall begin an analysis of system (3) with the simplest case where the fluctuations are weak and can be ignored. In this case, the slug flow is a medium with the effective average parameters, and system (3) takes the form

$$\frac{\partial P(z,L)}{\partial L} = -iP(z,L)\Big[-k_1\frac{\langle\rho_0\rangle}{\rho_1} + \frac{k_1}{2}\Big(\frac{\langle\rho_0\rangle}{\rho_1} - \rho_1 c_1^2\Big\langle\frac{1}{\rho_0 c^2}\Big\rangle\Big)P(L,L)\Big],$$
$$\frac{dP(L,L)}{dL} = -2ik_1\frac{\langle\rho_0\rangle}{\rho_1}(1 - P(L,L)) - \frac{ik_1}{2}\Big[\frac{\langle\rho_0\rangle}{\rho_1} - \rho_1 c_1^2\Big\langle\frac{1}{\rho_0 c^2}\Big\rangle\Big]P^2(L,L), \tag{4}$$
$$P(L_0, L_0) = \frac{2z_3}{z_1 + z_3}.$$

A solution of (4) can be found directly; however, it is possible to obtain it in a simpler way using the original system (2), because the medium's effective parameters — the sound velocity $1/c_m^2 = \langle\rho_0\rangle\langle 1/\rho_0 c^2\rangle$ and the acoustic impedance $z_m = \langle\rho_0\rangle c_m$ — can be determined immediately. After that, the problem represents a classical example from the theory of wave propagation in stratified media [9].

The formulas $a(L) = \exp(+ik_m L)(1/2)[(1 - z_m/z_1) + (1 + z_m/z_1)R]$ and $b(L) = \exp(-ik_m L)(1/2)[(1 + z_m/z_1) + (1 - z_m/z_1)R]$ describe the wave amplitudes a and b in the area of two-phase flow $P(z, L) = a(L)\exp(-ik_m z) + b(L)\exp(ik_m z)$, while the amplitude of the reflected wave is

$$R(L) = \frac{[(z_m - z_1)/(z_1 + z_m)] + [(z_3 - z_m)/(z_3 + z_m)]\exp[2ik_m(L - L_0)]}{1 + [(z_m - z_1)/(z_m + z_1)][(z_3 - z_m)/(z_3 + z_m)]\exp[2ik_m(L - L_0)]}.$$

The most typical case, where media 1 and 3 are the same fluid in which a train of slugs propagates, deserves consideration. In this case, $z_1 = z_3 = \rho_f c_f$ and then

$$R(L) = -\Big(\frac{1 - z_m/z_1}{1 + z_m/z_1}\Big)\{1 - \exp[2ik_m(L - L_0)]\}\Big/\Big\{1 - \Big(\frac{1 - z_m/z_1}{1 + z_m/z_1}\Big)\exp[2ik_m(L - L_0)]\Big\}. \qquad (5)$$

Let us draw a comparison with the known results, in particular, with the expression for the velocity of acoustic-signal propagation in a two-phase mixture of slug-train structure [4]: $c_m^2 = [\gamma P_0/\rho_f\varphi(1 - \varphi)]$, where P_0 is the static pressure and φ is the volume gas content of the mixture $[\varphi = l_g/(l_f + l_g)]$. Comparing this expression with

$$c_m^{-2} = \langle\rho_0\rangle\Big\langle\frac{1}{\rho_0 c_0^2}\Big\rangle = \Big(\rho_f\frac{l_f}{l_f + l_g} + \rho_g\frac{l_g}{l_f + l_g}\Big)\Big(\frac{1}{\rho_g c_g^2}\frac{l_g}{l_f + l_g} + \frac{1}{\rho_f c_f^2}\frac{l_f}{l_f + l_g}\Big),$$

we see that the coincidence is complete: $c_m^2 = c_g^2(\rho_g/\rho_f)(l_f + l_g)^2/l_f l_g$ for the most typical cases where $\langle\rho_0\rangle \approx \rho_f l_f/(l_f + l_g)$ and $\langle 1/\rho_0 c_0^2\rangle \approx (1/\rho_g c_f^2)(l_g/(l_f + l_g))$. However, the model used is also applicable to other situations, for instance, for description of multiple foams ($l_g \gg l_f$).

It should be noted that a similar expression for the sound velocity in a slug mixture for any gas content was obtained by the methods of acoustics of stratified media in [10]. This coincidence and the reference [10] were pointed out to us by the referee.

Since $z_m/z_1 \approx (c_m/c_f)(l_f/(l_f + l_g)) \ll 1$ (c_m is much less than the sound velocity both in the gas and in the fluid), the sound reflects from a train of slugs as from an absolutely soft medium except for the cases where an integer number of half-waves is contained in the area occupied by two-phase flow and the medium's bleaching takes place ($R \to 0$). It should be stressed that we deal with the wavelengths in the slug-flow area, in which $\lambda_m = (c_m/c_f)\lambda_1 \ll \lambda_1$ ($\lambda_1 = 2\pi/k_1$).

The Einstein–Fokker–Planck Equation (EFP). In the general case where the fluctuations are significant, we should determine the mean and correlation characteristics of the pressure field on the basis of the system of stochastic equations (3). Attempts to average (3) directly will lead to a linked chain of equations for the moments. In view of this, one proceeds in a different way in finding the statistical characteristics of boundary-value problems [8, 11]. For the corresponding Cauchy problem, one constructs a linear Liouville equation whose averaging does not involve serious difficulties for simple fluctuation models (the random telegraphic process is among them).

It can be easily verified [8] that $\varphi_L(U, U_L) \equiv \delta(P(z, L) - U)\delta(P(L, L) - U_L)$ satisfies the Liouville equation as a function of the variables L, U, and U_L. Since $P(z, L)$ and $P(L, L)$ are complex-value functions, $\varphi_L(U, U_L)$ depends, apart from L, on four arguments: either the amplitudes and phases or the real and imaginary parts. Recall that $P(z, L)$ and $P(L, L)$ are the solutions of system (3), which correspond to a definite realization of the process $s(z)$.

The probability density of realization of the solution of (3) is obtained by averaging $\varphi_L(U, U_L)$ over an ensemble of random quantity $s(z)$: $W_L(U, U_L) = \langle\varphi_L(U, U_L)\rangle$. The initial condition for $W_L(U, U_L)$ has the form [8] $W_L(U, U_L)|_{L=z} = \delta(U - U_L)\langle\varphi_z(U_z)\rangle(P_L(z, L)|_{L=z} = P(z, z))$. In view of the closure of the Riccati equation for $P(L, L)$, the Liouville equation for $\varphi_L(U_L) = \delta(U_L - P(L, L))$ can be obtained independently. In this case, the initial condition for the probability density of the field distribution at the boundary $W_L(U_L) = \langle\varphi_L(U_L)\rangle$ has the form $W_L(U_L)|_{L=L_0} = \delta(U_{L_0} - 2z_3/(z_1 + z_3))$. Below, we shall restrict our consideration to the case where $z_1 = z_3$ and $W_L(U_L)|_{L=L_0} = \delta(U_{L_0} - 1)$.

According to [11], we shall express $P(L, L)$ in terms of the reflectance $R(L, L) = P(L, L) - 1$ and obtain an EFP equation for the probability density of the reflectance-amplitude and phase distribution $W_L(\rho, \chi)$ $[R(L, L) = \rho_L\exp(i\chi_L)]$ for reflection from the two-phase medium of slug structure in the form

$$\frac{\partial W_L}{\partial L} + \frac{\partial}{\partial\rho}(AW_L) + \frac{\partial}{\partial\chi}(CW_L) = D\Big[\frac{\partial}{\partial\rho}B + \frac{\partial}{\partial\chi}F\Big]^2 W_L, \quad D = \frac{4l_f^2 l_g^2}{(l_f + l_g)^3}. \qquad (6)$$

Here

$$A(\rho_L,\chi_L)=\frac{k_1}{2}\Big[\frac{\langle\rho_0\rangle}{\rho_1}-\rho_1c_1^2\langle\frac{1}{\rho_0c_0^2}\rangle\Big](\rho_l^2-1)\sin\chi_L;$$

$$B(\rho_L,\chi_L)=\frac{k_1}{4}\Big[\frac{(\rho_f-\rho_g)}{\rho_1}-\rho_1c_1^2\Big(\frac{1}{\rho_fc_f^2}-\frac{1}{\rho_gc_g^2}\Big)\Big](\rho_L^2-1)\sin\chi_L;$$

$$C(\rho_L,\chi_L)=2k_1\frac{\langle\rho_0\rangle}{\rho_1}-\frac{k_1}{2}\Big[\frac{\langle\rho_0\rangle}{\rho_1}-\rho_1c_1^2\Big\langle\frac{1}{\rho_0c_0^2}\Big\rangle\Big]\Big[2+\Big(\rho_L+\frac{1}{\rho_L}\Big)\cos\chi_L\Big];$$

$$F(\rho_L,\chi_L)=k_1\frac{(\rho_f-\rho_g)}{\rho_1}-\frac{k_1}{4}\Big[\frac{(\rho_f-\rho_g)}{\rho_1}-\rho_1c_1^2\Big(\frac{1}{\rho_fc_f^2}-\frac{1}{\rho_gc_g^2}\Big)\Big]\Big[2+\Big(\rho_L+\frac{1}{\rho_L}\Big)\cos\chi_L\Big].$$

This equation was obtained under the assumption that the statistical characteristics of the acoustic signals whose wavelength exceeds considerably plug and slug sizes change only slightly on these scales. In the approximation considered, all the specific features of the telegraphic process are taken into account in the diffusivity D.

The physical conditions for the validity of this diffusion equation correspond to the situation where the wave has already undergone a sufficiently large number of collisions at the boundaries of slugs and plugs, and one can be satisfied with an averaged description of its behavior at greater distances in comparison with the sizes of individual plugs and slugs.

To solve (6), we shall use the following approximation [11], which permits us to simplify considerably the EFP equation. The point is that with no regard for fluctuations, the reflectance changes on the scale equal to the half-wavelength ($\lambda_m/2=\pi/k_m$) of the perturbation propagating in the two-phase medium [see (5)], i.e., $R(L)$ is a rapidly oscillating function. On the other hand, since $\lambda_m\gg l_g$ and l_f, the statistical characteristics of the acoustic perturbation change only slightly on the wavelength; in analyzing these perturbations, one can average (6) over the oscillation period. The probability density itself, which is, by definition, a quantity averaged over the ensemble of realizations of the random quantity $s(z)$, will not change on this scale; therefore, only the coefficients of Eq. (6) are subject to averaging. As a result of this procedure, we obtain a simpler equation [8] with coefficients that depend only on $\chi'=\chi-(k_m,L-L_0)$:

$$\frac{\partial W_L(\rho,\chi')}{\partial L}=-\frac{\partial}{\partial\chi'}\overline{C(\rho)}W_L(\rho,\chi')-D\Big\{\frac{\partial}{\partial\rho}(\overline{B'_\rho B}+\overline{B'_{\chi'}F})+\frac{\partial}{\partial\chi'}\overline{F'_\rho B}\Big\}W_L(\rho,\chi')$$
$$+D\Big\{\frac{\partial^2}{\partial\rho^2}\overline{B^2}+\frac{\partial^2}{\partial\chi'^2}\overline{F^2}\Big\}W_L(\rho,\chi'). \quad (7)$$

Here we used the fact that $\overline{A}=0$ and $\overline{BF}=0$. The bar denotes averaging over the oscillation period.

Further simplification consists in integration of (7) over χ', which produces the EFP equation for $\overline{W_L}(\rho)$:

$$\frac{\partial\overline{W_L}(\rho)}{\partial L}=-D\frac{\partial}{\partial\rho}(\overline{B'_\rho B}-\overline{B'_{\chi'}F})\overline{W_L}(\rho)+D\frac{\partial^2}{\partial\rho^2}\overline{B^2}\,\overline{W_L}(\rho),$$

or, writing the coefficients explicitly, we obtain

$$\frac{\partial\overline{W_L}(\rho)}{\partial L}=-\tilde{D}\Big[\frac{\partial}{\partial\rho}(\rho^2-1)^2\frac{\partial}{\partial\rho}+\frac{\partial}{\partial\rho}\frac{(\rho^2-1)^2}{\rho}\Big]\overline{W_L}(\rho),\quad \tilde{D}=k_1^2\frac{l_f^2l_g^2}{8(l_f+l_g)^3}\Big(\frac{\rho_fc_f^2}{\rho_gc_g^2}\Big)^2.$$

Since this equation differs from that considered in [8, 11] only by the form of diffusivity, we shall give its solution immediately.

It is convenient to change over to the variable $u=(1+\rho^2)/(1-\rho^2)$. For the distribution-probability density u, we have [8]

$$W_L(u) = \frac{\exp[-\tilde{D}(L-L_0)/4]}{2\sqrt{2\pi}[\tilde{D}(L-L_0)]^{3/2}} \int_u^\infty dx \, \frac{x\exp[-x^2/4\tilde{D}(L-L_0)]}{\sqrt{\cosh x - \cosh u}}. \tag{8}$$

The mean values are calculated directly using the solution obtained. The square of the reflectance modulus is described by the expression

$$\langle |R(L)|^2 \rangle = 1 - 4\pi^{-1/2} \exp[-\tilde{D}(L-L_0)/4] \int_0^\infty dx \, \frac{x^2 \exp(-x^2)}{\cosh\left(x\sqrt{\tilde{D}(L-L_0)}\right)},$$

whereas using the relation $\langle |P_{tr}|^2 \rangle = 1 - \langle |R(L)|^2 \rangle$, one can also determine the transmissivity through the two-phase flow layer. Let us give the asymptotic expression [12] $(L-L_0)\tilde{D}/4 \gg 1$ for $\langle |P_{tr}|^2 \rangle \approx 0.5\pi^{5/2} \exp[-(L-L_0)\tilde{D}/4](4/\tilde{D}(L-L_0))^{3/2}$.

The transmissivity index $\tilde{D}(L-L_0)$ is related closely to the Lyapunov index δ ($\delta = \tilde{D}/4$), which was the subject of the numerical calculations presented in [5]. Comparison of these quantities shows good agreement both in the functional dependence $\tilde{D} \sim \omega^2$ (for the long-wave regime ($\lambda_m \gg l_f$ and l_g) in the Sornette–Legrand classification scheme [5]) and in the order of magnitude.

Unfortunately, a direct comparison with the experimental data [4] does not seem possible for the following reasons. First, the fast-oscillation approximation is inapplicable for the reflectance under the conditions of [4], because for a perturbation duration $\tau \sim$ 15–500 msec, the wavelengths $\lambda_m \approx c_m \tau$ turn out to be comparable with the length of the working section of shock tubes (0.8 and 2.5 m).

Second, which is more important, the difference in the impedance characteristics of the fluid and of the gas is so significant that scattering by fluctuations remains strong for these wavelengths. Thus, under the conditions of [4]: $\tau = 20$ msec, $l_g = 0.04$ m, and $\varphi = l_g/(l_g + l_f) = 0.3$, the diffusivity $\tilde{D}$, which has the dimension of inverse length, is of the order of 3000 m^{-1} and exceeds l_f^{-1} and l_g^{-1} considerably. This indicates the inapplicability of the assumption that made the transition to (6) possible. The perturbation proves to be insufficiently long-wave for the character of its interaction with fluctuations to have the form of a diffusion process.

Stochastic Parametric Resonance in a Two-Phase Medium of Slug Structure. We shall consider statistical characteristics of an acoustic field inside a two-phase medium. As we have already verified, the procedure for averaging over a fast variable is very efficient for wavelengths l_f and $l_g \ll \lambda_m$, where $\lambda_m \ll 1/\tilde{D}$. The last condition means that fluctuations of the acoustic characteristics of the medium are, in a definite sense, small. However, not all spatial harmonics of these fluctuations play the same role. We shall take advantage of a formal analogy between Eq. (3) for reflectance and the equation of a linear oscillator with fluctuating parameters, in which k_m^2 plays the role of eigenfrequency [12]. Since the perturbation is small, we can assume, as is assumed in the theory of parametric resonance, that wave propagation in a two-phase medium is affected considerably by fluctuations either with short wavelengths ($q \ll k_m$) or with the wave vectors $\pm 2k_m + q$. The existence of resonant configurations will lead to the occurrence of wave-field characteristics growing deep in the slug flow. Let us strengthen the above reasoning by formal calculations.

It turns out to be possible [8] to express such a characteristic as the sound-field intensity $I(z, L) = P^*(z, L)P(z, L)$ in terms of the reflectance R and thereby to use to a considerable extent the results of the previous section. Indeed, from the first equation of system (3) we obtain the following integral representation for $I(z, L)$:

$$I(z, L) = I(z, z) \exp\left\{ -i \int_z^L \frac{k_2(z_2^2 - z_1^2)}{2z_1 z_2} (R(L') - R^*(L'))\, dL' \right\},$$

and from the second equation we obtain the integral identity

$$\frac{(1-|R(L)|^2)}{(1-|R(z)|^2)} = \exp\Big\{-i\int\limits_z^L \frac{k_2(z_2^2-z_1^2)}{2z_1z_2}(R(L')-R^*(L'))\,dL'\Big\}.$$

As a result, we have $I(z,L) = I_0(1+R(z))(1+R^*(z))(1-|R(L)|^2)/(1-|R(z)|^2)$, where I_0 is the intensity of the incident wave, and the asterisk refers to complex conjugation. As above, it is convenient to go from the variable ρ to $u = (1+\rho^2)/(1-\rho^2)$ and express explicitly the dependence on the rapidly varying phase χ_z:

$$I(z,L) = 2I_0\big[u_z + \sqrt{u_z^2-1}\cos\chi_z\big]/(1+u_L).$$

Since the effect we analyze occurs at distances much longer than the wavelength, it is expedient to consider only the slowly varying part of $I(z,L)$, i.e., to change over to the phase-averaged quantities denoted by the subscript a. Thus [8], $I_a(z,L) = 2I_0u_z/(1+u_L)$, $I_a^2(z,L) = 2I_0^2(3u_z^2-1)/(1+u_L)^2, \ldots, I_a^n(z,L) = g_n(u_z)/(1+u_L)^n$, where $g_n(u)$ is the nth-degree polynomial in u.

The intensity moments can be found in quadratures using the solution of the EFP equation (8). However, since the corresponding calculations involve a very peculiar technique (Møller–Fock transform), we shall give only the final result:

$$\langle I_a^n(z,L)\rangle = \pi\exp[-\tilde{D}(L-z)/4]\int\limits_0^\infty d\mu\mu\frac{\sinh\mu\pi}{\cosh\mu\pi}K_n(\mu)\exp[-\mu^2\tilde{D}(L-z)]\times\int\limits_1^\infty du g_n(u)P_{-1/2+i\mu}(u)W_z(u).$$

Here $W_z(u)$ is the solution of (8) for $L = z$, $P_{-1/2+i\mu}(u)$ is the Legendre function of the first kind, and $K_n(\mu) = \cosh\pi\mu\pi^{-1}\int\limits_1^\infty dx(1+x)^{-n}P_{-1/2+i\mu}(x)$. Without loss of generality, we made the origin of coordinates coincident with the medium's left boundary $L_0 = 0$.

Let us describe the asymptotic behavior of the intensity $L\tilde{D} \gg 1$. In this case, the regularities of variation in $\langle I_a^n\rangle$ will be of a general character, while the specific character of slug flow consists in the condition of validity of the above inequality. The spatial distribution of the intensity and its higher moments differ entirely. Thus, according to [13],

$$\langle I_a\rangle = \theta\Big(\frac{z}{L}-\frac{1}{2}\Big)\begin{cases} 0 & (z/L<0.5),\\ 0.5 & (z/L=0.5),\\ 1 & (0.5<z/L).\end{cases}$$

The more accurate estimate [14] shows that the transition zone is $\sim L^{1/2}$ in size.

For higher moments, the medium's layer is divided into three areas [8]: for $0 \leqslant z/L \leqslant 0.5(1-\sqrt{1-n^{-2}})$, the moments are exponentially small; for $0.5(1-\sqrt{1-n^{-2}}) \leqslant z/L \leqslant (1-1/(2n))$, they are exponentially large and reach the maxima with $z/L = 0.5\langle I_a^n\rangle_{\max} \approx \exp[\tilde{D}L(n^2-1)/4]$, and for $(1-1/(2n)) < z/L \leqslant 1$, they tend to unity by an exponential law.

In interpreting the behavior of the intensity moments, it should be noted that an increase in the higher moments in the central area does not imply by any means an exponential increase in the energy characteristics of the acoustic field. The energy flux

$$J = \frac{1}{4}(Pv^* + vP^*) = \frac{1}{2}\frac{(1-|R(z)|^2)\,I(z,L)}{(1+R(z))(1+R^*(z))z_1}$$

is an integral of motion, and for every realization, this flux is a constant in the entire slug-structure area.

The growth of moments corresponds to the existence of intensity overshoots, which is confirmed by the numerical-simulation results [8]. This circumstance can prove to be very important and lead to gas-slug collapse when a wave of not very high intensity is incident on the medium.

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